Quantum Computing
Problem Set 4

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Problem 1: Quantum teleportation via Bell measurements

We here consider a teleportation protocol that involves three qubits, where the sender Alice holds qubits 1 and 2 and a very remote receiver Bob holds qubit 3, just as in the version discussed in the lectures. Also the initial state is assumed to be the same as in the lectures

$$|\phi(t_0)\rangle = |\psi_1\rangle \otimes \frac{1}{\sqrt{2}}(|0_2, 0_3\rangle + |1_2, 1_3\rangle) \tag{1}$$

where $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ is an arbitrary state to be teleported to qubit 3.

The protocol we discuss here however differs from the one discussed in the lectures in the operations that Alice and Bob perform.

a) In the first step, Alice will measure qubits 1 and 2 jointly in the so called Bell basis. That is, she will treat qubits 1 and 2 as one quantum system, for which the four Bell states,

$$|B^a_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_1, 0_2\rangle + |1_1, 1_2\rangle) \tag{2}$$

$$|B^b_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_1, 0_2\rangle - |1_1, 1_2\rangle) \tag{3}$$

$$|B^c_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_1, 1_2\rangle + |1_1, 0_2\rangle) \tag{4}$$

$$|B^d_{1,2}\rangle = \frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle) \tag{5}$$

form a basis, that is called the *Bell basis*, i.e. $\langle B^\mu_{1,2}|B^\nu_{1,2}\rangle = \delta_{\mu,\nu}$. Assuming there is an operator that is diagonal in this basis and has different eigenvalues for all Bell states, measuring this operator will project the system into one of the Bell states $|B^a_{1,2}\rangle, |B^b_{1,2}\rangle, |B^c_{1,2}\rangle$ and $|B^d_{1,2}\rangle$. Here we are not interested in the eigenvalues of this operator but only in the probabilities for each outcome. Hence calculate the probabilities for projecting the system into one of the Bell states $|B^a_{1,2}\rangle, |B^b_{1,2}\rangle, |B^c_{1,2}\rangle$ and $|B^d_{1,2}\rangle$ in this measurement for the state $|\phi(t_0)\rangle$ as given in Eq. (1).

b) Depending on the outcome of the above Bell measurement, which Alice communicates to Bob, Bob needs to perform some operations on qubit 3 to bring it into the state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$. Which are these?

c) Assume Alice doesn't tell Bob the outcome of her measurement. Can Bob do anything to his qubit to generate the state $|\psi\rangle$?

d) Now assume that qubit 1 is initially entangled with another qubit, qubit 0. Hence instead of qubit 1 being in the state $|\psi\rangle$ we have the entangled state

$$\alpha |0_0, 0_1\rangle + \beta |1_0, 1_1\rangle) \tag{6}$$

for qubits 0 and 1. What is the state of qubits 0 and 3 at the end of the protocol? What happened to the entanglement?

e) Can the protocol also teleport a mixed state?

Problem 2: Probabilistic entanglement generation using postselection

Consider two qubits in the product state

$$|\phi_{1,2}\rangle = (\alpha_1 |0_1\rangle + \beta_1 |1_1\rangle)(\alpha_2 |0_2\rangle + \beta_2 |1_2\rangle) \tag{7}$$

Can you think of a measurement that would leave the two qubits in an entangled state after the measurement?